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Who I am

Pour l'épisode en français, cliquez ici ⇨ Les maths comme je les aime … et comme je les raconte ! 🙂

All episodes published : *https://www.mathasilikeit.com*

Math as I like it /4

The PLANE, an "invention" you can't bypass!

You can't bypass the plane because it is one of the three basic elements of Euclidean geometry (and, 22 centuries later, Hilbert's geometry): the point, the line, the plane! What I wrote about the line is just as true for the plane: without it, Euclidean geometry would not get very far! (Without the point either, of course, but if after all the previous episodes you're not convinced, I give up!)

But there is another reason why you can't circumvent the plane... I'll come back to that soon. 😊

Right, shall we start at the beginning? To invent the plane, we're going to need lines (that was episode 3)... and surfaces. I haven't told you anything about surfaces yet, and I don't really want to devote a whole episode to them, so would you mind if we just had a paragraph 0 in this episode? Here we go!

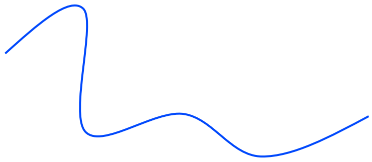
– Hey, why are you asking our opinion if you're not even waiting for our answer? But yes, okay, we're fine with that!

– Thank you... and you're right, I should have waited! I always want to rush things!

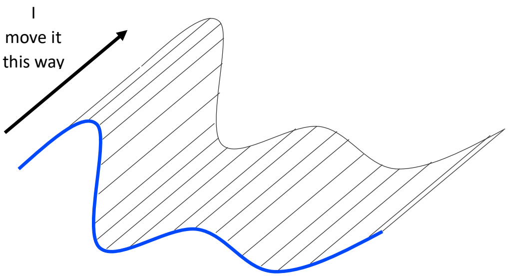
What is a surface?

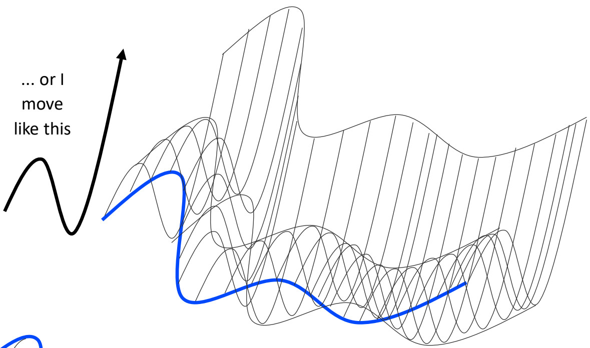
Do you remember (*episode /2*) that a curve is a set of points: the path of a point object?
Sorry, I'm repeating myself. But also, history repeats itself so often: **a surface is the path of a *curve* object**!
(Like curves, surfaces are sets of points, therefore places.)

In episode /3, I created a curve object and moved it so that it pass in several different ways through two points I had chosen: what interested me was not how it moved but only its starting and ending positions.

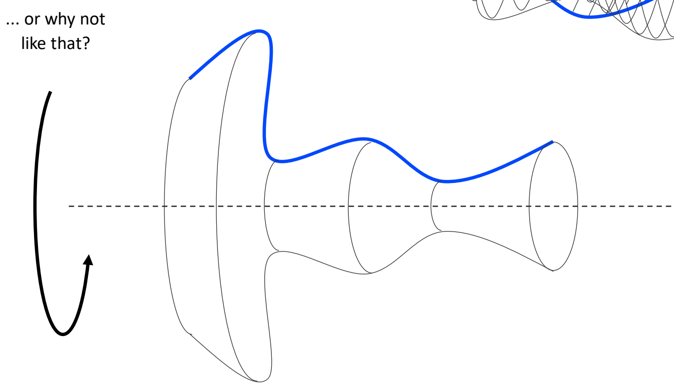


But today I am going to focus on all the points it passes through during its movements (to visualize this set, imagine that the curve object "leaves a trace" at each of these points):





That's what a surface is!



*As you know, here **I talk about math**:*

so I'm not trying to rigorously define the concept of surface (some surfaces are really very annoying, cannot be generated by the movement of a curve object)...

just to give a (superficial☺) idea of what it is!

"Inventing" the plane.

A plane is a surface: the set of points crossed by an *usco* (an *unlimited straight curve object*, you remember what that is, right? If not... go back to episode /3😊) to which we impose a carefully chosen movement.

What movement? In fact, there are several possibilities: most of them require several uscos... and what's more, they end up escaping! It is possible to make do with a single usco – and not lose it – but that complicates the drawings a bit, so I'm going to suggest the most traditional option: it also uses the most uscos (it loses 4!), but it's really clear. Okay?

– Um, are you really waiting for our answer? That's good! So yes, okay... but can you still show us quickly how you do it with just one usco – and without even losing it?

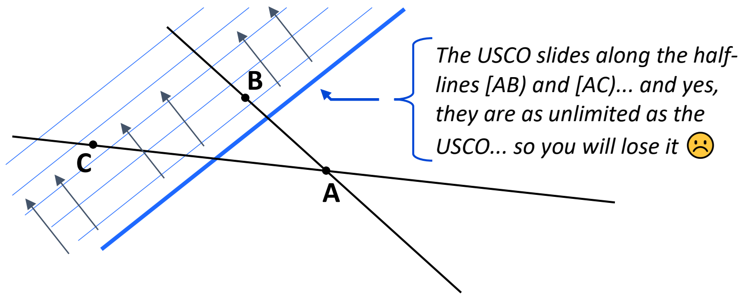
– You see, I'm listening! Okay, but quickly, just the principle!

Here we go with the traditional method:

you choose three points in space (I'll call them A, B, and C) and imagine the lines (AB) and (AC). I'll draw them in black:

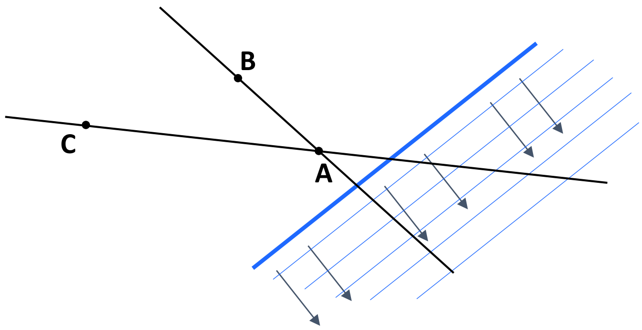
Then you move an *USCO* in such a way that it remains in contact with 2 of the 4 original half-lines A

like this (imagine it sliding as if on two rails):



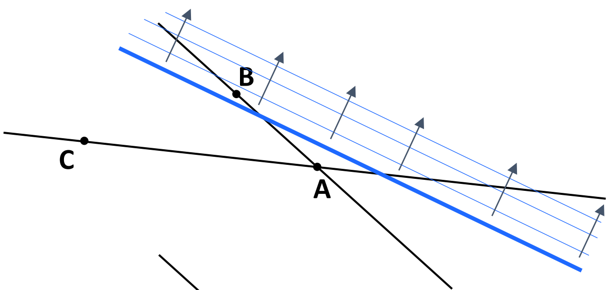
Then a 2nd *USCO*, like this:

(*and of course you'll lose it* ***too*** ☹)



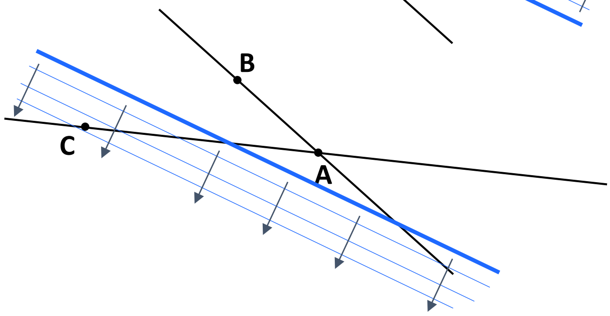
... a 3rd one, like this:

(*... just like this one* ☹)

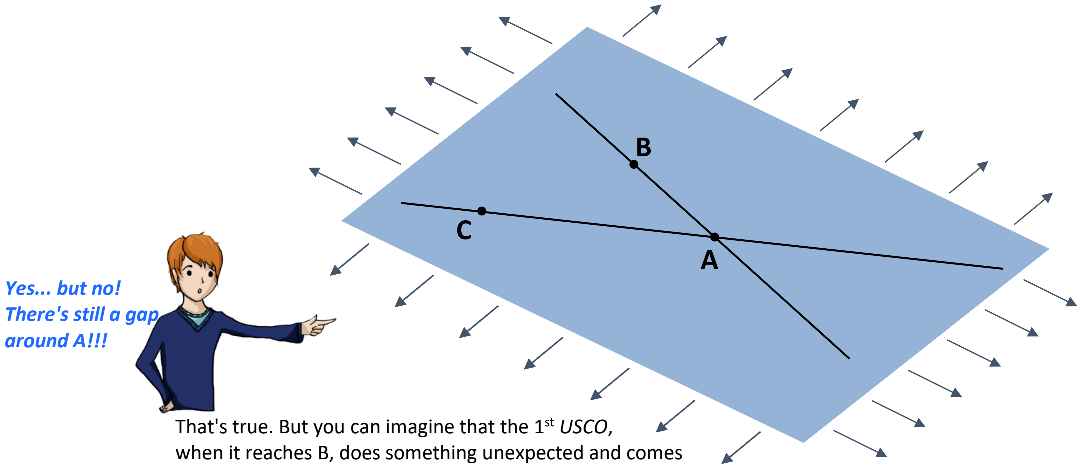


... and finally a 4th, like this:

(*... and like this one* ☹)



But ultimately, if you remember that a line is unlimited (and so are USCOs), the points crossed by your four uscos on their way to infinity form an immense surface, a surface without any dips or bumps: **a plane**!



That's true. But you can imagine that the 1st *USCO*, when it reaches B, does something unexpected and comes back to sweep a part of the half-line [CA), pivoting around B, like *the* red *USCO*. Just enough to fill the gap...
Then it returns to its initial position and continues on its way.

No dents or bumps? It's easy to "see," but proving it is a little more complicated. But here, I'm just telling the story, I'm not proving anything, right? So it doesn't really matter😊 !

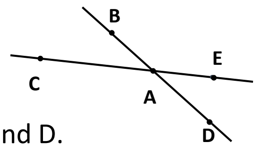
– Okay... and the other method, the one with a single USCO? An USCO that you don't lose? All those lost USCO's, it's a bit sad, isn't it?

– *You certainly are consistent in your ideas! But really quickly, there are still one or two things I'd like to tell you. And I'm not going to shade in the areas crossed by the usco, just show you its movements.*

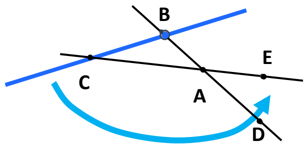
Here we go! The plan, version 2:

Start with the three points A, B, and C that you already know,
imagine the lines (AB) and (AC) and two new points:

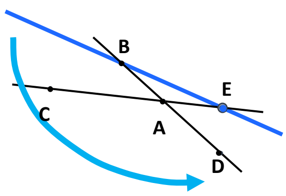
D on (AB) and E on (AC), such that A is between C and E, and between B and D.



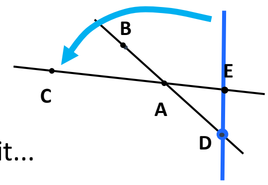
Place your *USCO* so that it passes through B and C,
then, without leaving B (it pivots on it), slide it from C to E
along the entire segment [CE].



Then, without leaving E, drag it from B to D,
all along the segment [BD]



... and finally, without leaving D, drag it from E to C
all along the segment [EC]



There you go. Now you can put your *USCO* away, you haven't lost it...
And it has generated the same plan as the previous USCO.

Okay, we still have a few minutes left, so for those who are hooked (if there are any), let's take a quick look at a very different method:

I could have tried to extend the principle of my definition of a line to the definition of a plane.

A plane would then be the set of points crossed by a particular *surface* object which, regardless of how it moves without leaving points A, B, and C, would always occupy exactly the same place in space.

This definition would have had the advantage of clearly showing that three (non-aligned) points define a plane.

I did not do this because, while a line is the only curve object that can "slide" through two chosen points while maintaining exactly the same position in space, a plane is not the only surface object that can slide through three chosen points A, B, and C without changing position: the sphere, which has as its "great circle" the circle circumscribed around triangle ABC, also has this particular property.
(The problem was already apparent with the line: given two points A and B, the unlimited straight line is not quite the only curve object that, regardless of how it occupies A and B, does not change location. Why? Because this is still true for a curve object that occupies exactly the segment [AB]: it has no choice anyway, it cannot move! I got around this difficulty by requiring my straight curve object to *pass through* A and B – to "slide" through them😊)

End of the minute-for-addicts... and of the episode.

– Oh no, aren't you forgetting something? What you wrote at the very beginning:

> *"But the plan is essential for another reason as well... I'll come back to that soon."*

– Oops! You're right! So, a few more lines:

The plane is doubly essential. For its dominant position in geometry, as I have already mentioned... but also for its position in the universe, which is rather frightening: in math class, everyone finds it commonplace to say "I am drawing plane $\mathscr{P}$"... or "observe the plane that passes through point A and line d," for example. But what we draw – or what we observe – is only a tiny part of a plane:

A plane divides our universe in two. Our entire universe: if you are on one side of this plane and want to get to the other side, you have no choice but to cross it. A plane is *physically* unavoidable.

– But why is that scary?

– Yes, well, maybe I got a little carried away there, because a plane is just a place. But just imagine for a second that you could be forbidden from crossing it? Fortunately, other than by magic, that seems rather unlikely... and although mathemagique.com publishes my books, I never taught at Hogwarts 😊