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Les maths comme je les aime ... et comme je les raconte ! 😊

All episodes published : <https://www.mathaslikeit.com>

Math as I like it /3



... after that, it's straight ahead!

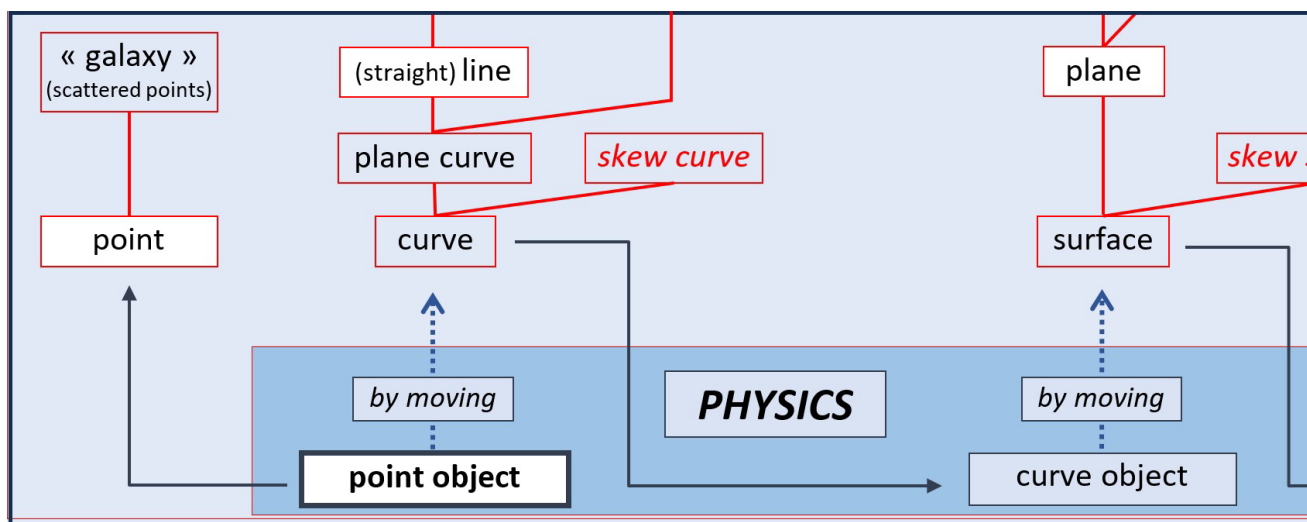
Why have I focused so much on the point object, the point, the curve? Because they are the basic elements of geometry, of course... but especially because they are the "parents" of the (straight) line. And without the (straight) line, Euclidean geometry would not go very far! What did you say? "The line is easy, it's a curve that goes straight ahead"?

Sigh: first of all, a curve is a place, it doesn't go anywhere... and secondly, what does it mean for an object to "go straight ahead"? In our physical universe, apart from photons, there aren't many objects that always go straight ahead (and even photons can deviate)!

Inventing the (straight) line?

Well, this isn't new; geometry *imagines!*

It is based on a set of abstractions, idealizations of our physical universe:



(An excerpt adapted from the tree structure of "so, according to": <https://donc-dapres.com/>)

So, the (straight) line?

[Okay, okay! From now on, I won't put that "straight" in parentheses anymore. I promise! Why was it there? Because I can't help thinking in French. Reread the last page of episode zero?]

Let's go! Well, almost. I'm still missing a little bit of abstraction, a new imaginary object, *the curve object* (there, at the bottom right of the screenshot! 😊).

In short... one last step before reaching the right:

-) what is a curve object?

You obviously remember that a curve is a set of points: the path of a moving point object?

Now imagine that as the point object moves, it drags a very thin thread behind it... a really, really thin thread! Even thinner than that! A thread *as thin as a point*.

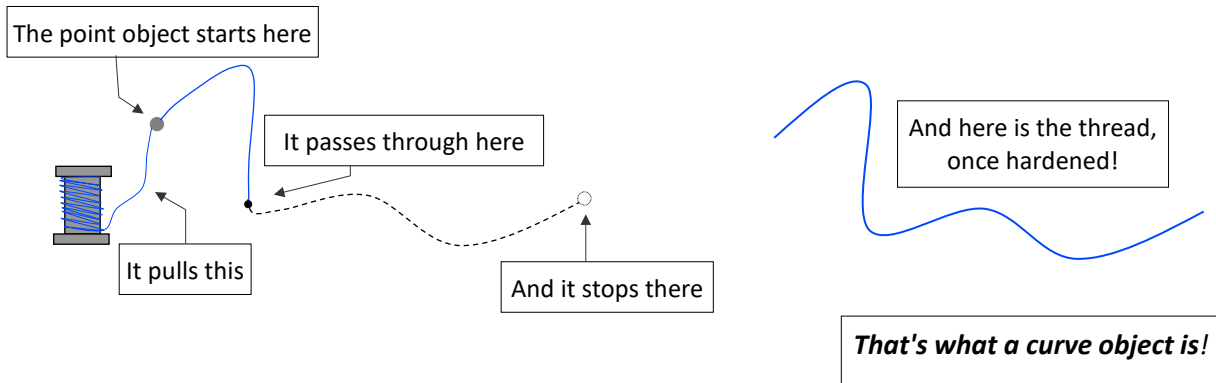
An **imaginary** thread, of course.

This thread materializes the path of the point object: it occupies all the points crossed by it.

What if, after a few seconds, this thread hardened and became rigid?

You would have an object in the shape of a curve – and *no bigger than a curve!*

An object that you could, in turn, move:



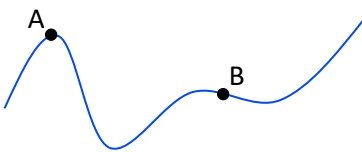
You already had the point object, now you have a curve object!

What are you going to do with it? Move it, of course 😊

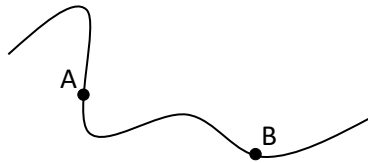
-) From the curve object to the line.

Mark two points, A and B, on a sheet of paper. You want to place your curve object so that it *crosses* these two points. You can do this in many different ways,

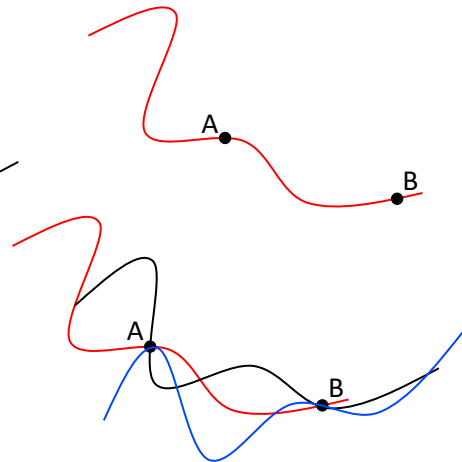
for example like this... :



... like this... :



... like this... :



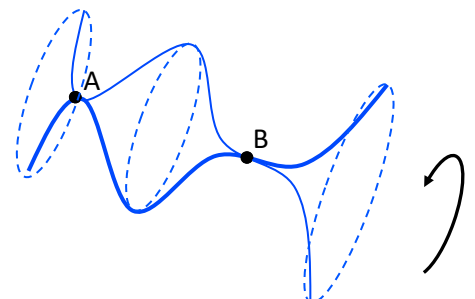
If I combine these three methods I get a very nice drawing... which highlights three *different* sets of points: the points occupied by the curve object depend on its position!

And it's even clearer "in space":

after all, we live in a three-dimensional space (at least 😊), we don't slide on a sheet of paper... so *imagine* that points A and B are floating in space in front of you and that, using some technique, you have made your curve object *pass through them* (and, just to simplify the drawing, always at the same points on the object). Then, if you give it a gentle push (a curve object is fragile!), it will spin like a top and occupy hundreds of different curves, one after the other:

I wrote "hundreds" so as not to alarm anyone but you know that it's actually the same story as for the question "how many points are there on a curve between two points on that curve?" in the previous post (do you draw a curve or a stroke?):

your curve object will not occupy a few hundred curves in succession, but an infinite number!



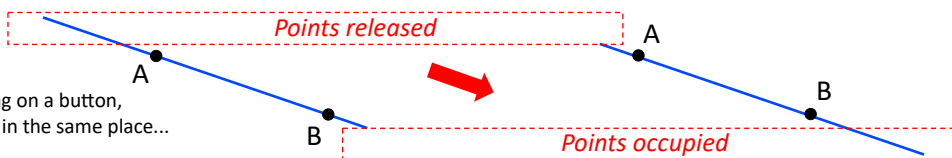
Well, now **we are going to imagine a curve object unlike any other**, a curve object that, when "trapped" by two points in space that it must constantly *cross*, can struggle as much as it wants, slide or twirl, it will always occupy exactly the same curve!

This particular curve object, this hyper-thin thread, cannot have any hollows or bumps, otherwise, as in the drawing above, it could occupy an infinite number of curves, at least some parts of which would be different. Let's call it a "straight curve object" ("straight" as opposed to "bent" 😊), or "**sco**".

This *sco* cannot be limited either, otherwise when it slides through the two points it will release some of the points it occupied in favor of new points that it did not occupy before – it will therefore no longer occupy exactly the same curve:

This is obviously just a diagram:

there is only one point A, one point B and only one *sco* (in blue)! To be perfect, I should have created an animation that, when clicking on a button, would have made the first or second drawing appear in the same place... but I'm *not* perfect!



So, among all the curve objects that pass through two chosen points, the only one that – regardless of its position – always occupies exactly the same curve is an "unlimited straight curve object" ... an "**usco**"!

And what about the line in all this? No, don't tell me it's not obvious now:

the line defined by the two points A and B is the curve occupied by *the usco* that passes through these two points...

and no, I won't draw it because it's unlimited 😊 !!!

* * * * *

A first clarification: *like all curve objects, the usco is imaginary – and its name is not, absolutely not, official. But I liked it, so I kept it.*

A second clarification: *the different versions of Euclid's axioms define the segment **and then** extend it into a straight line. I tend to go from the general to the specific (and I'll do it again soon with sets of integers)... but this very personal interpretation of math is really just that, a colorful interpretation, for educational purposes, of [Euclid's and Hilbert's axioms](#).*

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