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[Who I am](#)

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Les maths comme je les aime ... et comme je les raconte ! 😊

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Math as I like it /2



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Are you drawing a curve or a stroke?

Well, I think I've emphasized the point enough, haven't I?

But it's not just because I'm monomaniacal, it's also because it's at the origin of our Euclidean geometry. I wrote this in the first "journey" of "[donc, d'après](#)" (so, according to):

to begin our geometry, I had to invent either the point object or the point... the other follows quite simply. I chose to invent the point object, because it seems easier to me to imagine the reduction of an object than that of a place.

Now that we know what a point object is – and what a point is – we can move on to the next question: **what is a curve?** (Nooo, it's not a stroke, any more than a point is a stain!)

Remember [loran](#) and [Apo](#), the wizard and the Atlantean who became point objects?

When they move, they pass through a set of points: that's what a curve is!

A curve is the path of a moving point object (the set of points it passes through: curves are not always straight).

[I can already hear a question from the back: [what is a set?](#)]

Okay: think about the page you're reading right now, the time it is, and your best friend:

your thought may be the only link between these three objects(*)... but it gives you an idea of what a set is.

This is just an image, of course: a set is like a thought about objects. You can be interested in bizarre sets by bringing together, as I have done, objects that don't have much in common – or you can observe sets whose objects are all of the same type: for example... sets of points 😊 !

(*) In the mathematical sense of the word "object," which is broader than the physical sense: pretty much anything that can be defined unambiguously. "Real" objects, of course, but also living beings and even the dream you had yesterday morning just before you woke up – even if you don't remember it anymore!]

A curve, therefore, is a set of points... which are places. And a set of places is also a place, so **a curve is a place**: you can occupy it or cross it, but you can't grasp it, move it... **or draw it** 😊 !

If loran or Apo – in bright mode – moved fast enough, you would see a bright curve, but of course that's not true: a single point does not shine. You would simply be experiencing retinal persistence; at any given moment, there would only be a single illuminated point (but we won't get into the "phi effect" here, as that would take us a bit too far afield)!

So what about a stroke? It's just a human construct that allows us to form an idea of a particular curve. A *very rough* idea, but we have no choice. However, if we want to "think correctly," it is absolutely necessary to keep in mind that a curve and a stroke are not at all the same thing: the family of curves is one of the major families of geometric classification (along with points, surfaces, and "solids" – which, despite their name, are still only places)... so it seems rather important to me to have a reasonable idea of this!

Two fundamental differences between a curve and a stroke:

-) **a curve is a place, a stroke is an object** – a collection of particles of matter:

Grate a pencil lead onto a sheet of paper:

you have drawn a stroke by depositing pencil dust between the fibers of the paper.

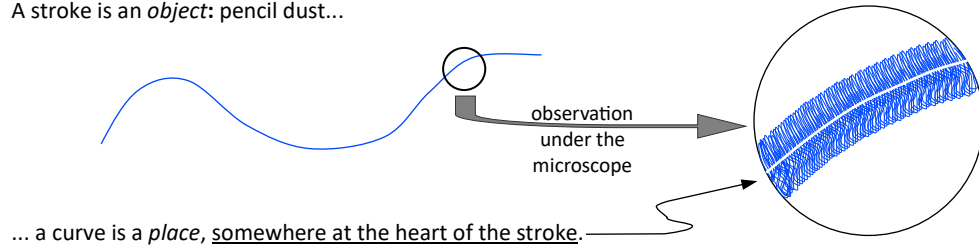
You could also:

soak the sheet with ink – using a pen to dye the fibers,
embed solid ink particles in it (dot matrix printers),
spray it with liquid ink (inkjet printers),
melt ink dust onto it, then dry it – photocopiers, laser printers, etc.

... or make a stroke appear on all kinds of media^(*), using all kinds of processes:
chemical (photos), optical (cinema, laser), electronic (television, monitors), etc.

(*) Not to mention the blackboard on which chalk is rubbed, leaving dust that settles – in part! – in the grooves of the board!

A stroke is an *object*: pencil dust...



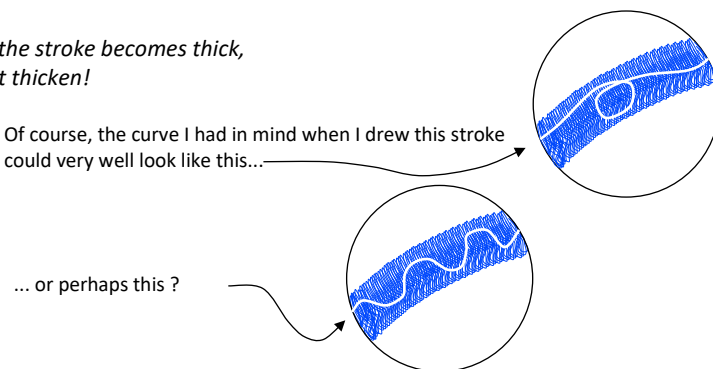
-) **A curve has no thickness** – or, which amounts to the same thing, it has the thickness of a point:
the point is "*smaller than small*," the curve is "*thinner than thin*".

... **a stroke is thick**: when you draw a stroke to draw attention to a curve, it is as if you were painting over the curve with a brush that is *infinitely* too wide for it. You cover the curve, of course, but you go over it, you smear all around it even with a pencil that *seems* very thin to you.

Under a microscope, *the stroke becomes thick*,
but the curve does not thicken!

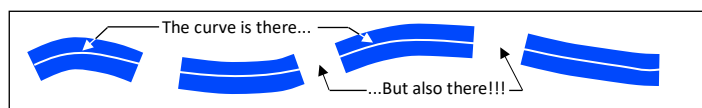
Of course, the curve I had in mind when I drew this stroke
could very well look like this...

... or perhaps this ?



But mathematicians decided that the strokes they drew should give as clear an idea
as possible of the curves they were representing. So let's keep it simple!!!

One last clarification: a dashed stroke does NOT represent a dashed curve. Or you have to specify it!



Two consequences of the differences between a curve and a stroke:

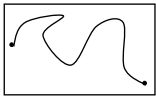
-) **A stroke is limited, a curve may not be:**

being a physical creation, a stroke begins in one place and ends in another.

The limit of a curve, on the other hand, depends only on our imagination:

it is up to us to decide that the space in which our point objects evolve is unlimited (it is therefore not necessarily the universe as we know it, but an idealization of that universe), it is also up to us to decide that these point objects we have invented...

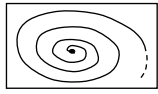
- can either move instantaneously (yes, I know... not in our physical universe!)
- or are eternal (yes, I know!)



We can "easily" conceive of a **limited curve**:

it is the path of a point object between two points - or of a point object that returns to its starting point (the curve is then "closed").

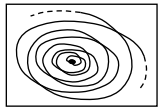
A line segment, an arc of a circle, a polygon, a circle, or even a path formed by three line segments followed by two arcs of circles (but we don't know all that yet!).



An unlimited curve is obviously more difficult to conceive:

for example, we need to imagine an *eternal* point object that sets out to explore our ideal universe without stopping. Not even in 5 billion years, when our sun will have stopped shining!

A half-curve or a simple spiral (which we still don't know!) are unlimited curves.



We can even imagine two point objects that start successively from the same point but in two different directions. We then conceive of an unlimited curve in both directions.

For example, a line, of course, but also a double spiral, and many other curves.

We can conceive, we can imagine... but admit that it *really* takes some imagination! That's what math is all about. 😊

-) **A curve is formed by an infinite set of points and a stroke by a finite set of particles:**

for a stroke, that seems reasonable (even if there can be a very large number of particles).

But what about the curve?

First of all, don't confuse "limited curve" with "finite curve":

limited means that the curve has two points that stop it... or that it is closed!

Finite means that you could count the points that make it up... count them all!

And that's not possible: even a limited curve is composed of an infinity of points!

How do we know that no curve is finite?

Easy: Choose a curve, then two points on that curve...

two *different* points – so two separate points (*if they weren't separate, it would mean that no microscope, even a super powerful one, could tell them apart. You would have just picked the same point twice*):

Is there at least one other point on this curve between the two you have chosen?

Yes, otherwise how would a point move from the first point to the second?

Good. You chose "point 1" and "point 2," and now you have "point 3," *between* "point 1" and "point 2"...

And now:

is there at least one other point on the curve between "point 1" and "point 3"?

I suppose you can guess what happens next?

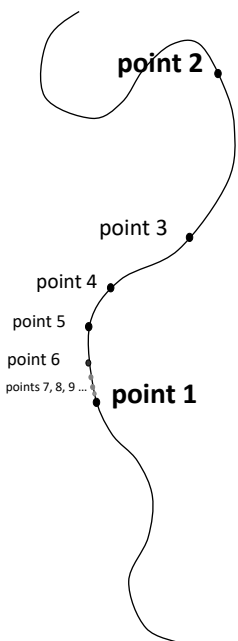
You end up with "point 4," "point 5," "point 6"...

and it will never stop: but NO, you will not end up back at "point 1," you could "verify" this by using increasingly powerful microscopes!

What if you've already used the most powerful microscopes?

That only proves the limits of our technology:

imagine a perfect microscope!



I think I've said this before: geometry is above all a matter of imagination 😊

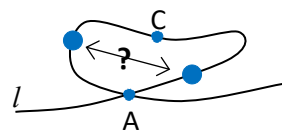
Perhaps you noticed the very discreet appearance of the word "between"? It's one of those words that seem very simple... until you dig deeper! And no, I'm not really going to dig deeper, just scratch the surface a little:

if A, B, and C are three points on a curve l , and if this curve l passes only once through A and C, then "B is a point on curve l , between A and C" means that:

a point object that moves from A to C, without leaving curve l , crosses B.




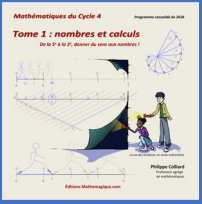
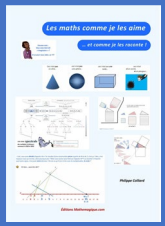
... but if curve l passes through A and C several times, "between A and C" no longer has any precise meaning:



Perhaps you have also noticed that nowhere in this episode do I insist on the idea that the space of points we imagine is complete? (the closest I come to this is in the creation, above, of "point 3": "Otherwise, how would a point object move from the first point to the second?")

A little patience, it will come, of course. But not just yet: it seems fundamental to me, from these first posts, to highlight the **at least** countable infinity of our space of points: as David Bessis kindly pointed out to me one day during an unexpected encounter, my "point objects" as I describe them (episodes [1](#), [1a](#), and [1b](#)) are **limit objects** – and points are also **limit points**, places that only point objects can occupy without overflowing!

Hilbert's axiom of linear integrity and the power of the continuum will come in due course... and quite naturally! But here we are still stammering, we need to prepare the ground for them: not saying everything at once, but also not saying anything false, not locking ourselves into a narrow view of our space of points!

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